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FORMULA SHEET

CAS-EXAM5

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FORMULAS

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TOPICS

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RATEMAKING DATA & EXPOSURES

17 items

Calendar year incurred loss

$L_{CY} = \text{Paid}_{\text{year}} + (R_{\text{end}} - R_{\text{begin}})$, Paid = losses paid during the year, R_end = ending case reserve, R_begin = beginning case reserve

Customer lifetime value closed form

$\text{CLV} = \frac{m}{1-r\,v}$, m = constant annual margin per policy, r = retention rate, v = annual discount factor

Transition-year claims-made rate from the cumulative report-lag step factor

$\text{Rate}_n = \left(\sum_{j=0}^{n-1} p_j\right) \times \text{Mature Rate}$, p_j = share of ultimate cost at report lag j, n = policy maturity year

Excess ratio for large losses

$\text{Excess Ratio} = \frac{\sum \text{losses above threshold}}{\sum \text{losses capped at threshold}}$, threshold = large-loss cap; loaded losses = capped losses $\times(1 + \text{Excess Ratio})$

Additive expense fee (policy fee) under the variable permissible loss ratio

$\text{Fee} = \frac{F}{1-V-Q}$, F = average fixed expense per exposure, V = variable expense %, Q = target profit %, denominator = variable permissible loss ratio

Off-balance factor for class relativity changes

$OB = \dfrac{1}{1+\Delta d}$, OB = off-balance factor applied to the base rate, Δd = change in the exposure-weighted average rate differential

Classical partial credibility square-root rule

$Z = \sqrt{\frac{n}{n_{\text{full}}}}$, Z = credibility weight, n = observed claim count, n_{full} = full-credibility standard (e.g. 1,082 claims)

Exponentially trended value over a trend period

$V_{\text{future}} = V_{\text{base}}\,(1+r)^t$, V_base = base value, r = annual trend rate, t = trend period in years (midpoint to midpoint)

Parallelogram on-level factor

$F = \dfrac{I_{cur}}{\bar{I}}$, F = on-level factor, I_{cur} = current cumulative rate index, $\bar{I}$ = area-weighted average rate index for the experience period

Bühlmann credibility factor

$Z = \frac{n}{n+K}, \quad K = \frac{\text{EPV}}{\text{VHM}}$, n = claim count, K = Bühlmann constant, EPV = expected process variance, VHM = variance of hypothetical means

Frequency-severity pure premium cross check

$\frac{C}{E} \times \frac{L}{C} = \frac{L}{E}$, C = claim count, E = earned exposures, L = incurred losses; frequency times severity must equal losses over exposures

Indicated rate change factor under the loss ratio method

$\text{CF} = \dfrac{(L+E_L)/P + F}{1-V-Q}$, L = losses, E_L = LAE, P = on-level premium, F = fixed expense ratio, V = variable expense ratio, Q = profit provision; percent change = CF − 1

Actuarially sound rate satisfying Principle 2 (all costs)

$R = \dfrac{PP+F}{1-V-Q}$, PP = pure premium (loss+LAE), F = fixed expense per exposure, V = variable expense ratio, Q = profit and contingency provision

Exposure trend factor for an inflation-sensitive base

$(1+g)^n$, g = annual exposure inflation rate (wage or price growth), n = trend period in years from historical midpoint to future policy midpoint

Indicated rate under the pure premium method

$R = \frac{PP+F}{1-V-Q}$, R = indicated rate per exposure, PP = pure premium (losses+LAE per exposure), F = fixed expense per exposure, V = variable expense %, Q = profit and contingencies %

GLM factor relativity under a log link

$R_j = e^{\beta_j}$, R_j = multiplicative relativity for the factor level, β_j = fitted log relativity (coefficient); base level has $\beta_j = 0$ so $R_j = 1.00$

Net cost of reinsurance load

$NC = \text{Ceded Prem} - E[\text{Recoveries}]$, NC = net cost loaded per exposure, Ceded Prem = reinsurance premium ceded, E[Recoveries] = expected ceded recoveries

Expected emergence over an interim period

$E_{\text{emerge}} = \frac{U-A_{t_1}}{1-\%_{t_1}} \times (\%_{t_2} - \%_{t_1})$, U = prior ultimate, A_{t_1} = actual emerged to date, $\%_{t_1}$ = percent emerged at last valuation, $\%_{t_2}$ = percent emerged at interim

Net unpaid claims from gross unpaid and unpaid recoveries

$\text{Net Unpaid} = \text{Gross Unpaid} - \text{Unpaid S\&S} - \text{Unpaid Reins}$, S&S = salvage & subrogation, Reins = reinsurance recoverable, all unpaid = ultimate minus received to date

On-level severity restated to a common cost level via an inflation index

$S_{\text{on-level}} = S_{\text{raw}} \times \frac{I_{\text{current}}}{I_{\text{claim year}}}$, S_raw = raw severity, I_current = index at target level, I_claim year = index at claim's cost level

Unpaid claim reserve from ultimate losses and percent paid

$R = U \times (1-p)$, R = unpaid claim reserve, U = ultimate losses, p = proportion of ultimate losses paid to date

Bornhuetter-Ferguson ultimate losses

$U_{BF} = \text{Reported} + \text{Premium} \times \text{ELR} \times (1-p)$, ELR = a priori expected loss ratio, p = 1/CDF = reported percentage, 1-p = unreported percentage

Unpaid ULAE under the classical paid-to-paid technique

$\text{Unpaid ULAE} = R \times [0.50 \times C + 1.00 \times I]$, R = paid-ULAE-to-paid-loss ratio, C = case reserves, I = pure IBNR (claims not yet opened)

Total ultimate losses with a separate large-claim provision

$U_{total} = L_{ult} + P_{large}$, U_{total} = total ultimate, L_{ult} = developed limited ultimate (limited losses × limited LDF), P_{large} = separately estimated large-claim provision

Berquist-Sherman adjusted reported losses for a cell

$R_{adj} = \bar{C}_{adj}\,(N - K_{adj}) + P_{adj}$, $\bar{C}_{adj}$ = adjusted avg case, N = reported counts, K_{adj} = adjusted closed counts, P_{adj} = adjusted paid

On-level premium factor from cumulative rate levels

$\text{OLF}_y = \frac{RL_{cur}}{RL_y},\ \text{OLEP}_y = EP_y \times \text{OLF}_y$, RL = cumulative rate-level index, cur = current year, y = accident year, EP = earned premium, OLEP = on-level EP

Disposal (claim settlement) rate at a given maturity

$d = \frac{C}{U}$, d = disposal rate, C = closed claim counts at the maturity, U = ultimate (or reported) claim counts

Implied average unpaid per open-plus-IBNR claim

$\text{Avg Unpaid} = \frac{\text{Case O/S+IBNR}}{\text{Open Counts+Unreported Counts}}$, Case O/S = case outstanding reserve, IBNR = incurred-but-not-reported reserve

Roll-forward reserve identity with prior-year development

$R_{current} = R_{prior} - P_{period} + \Delta U$, R = held reserve, P = paid during period, ΔU = new ultimate − prior ultimate (development)

Volume-weighted age-to-age development factor

$f_k = \frac{\sum_i C_{i,k+1}}{\sum_i C_{i,k}}$, f_k = factor from maturity k to k+1, C_{i,k} = losses for accident year i at maturity k

Cumulative development factor to ultimate including the tail

$\text{CDF}_{\text{ult}} = (\prod_k f_k) \times f_{\text{tail}}$, f_k = observed age-to-age factor at age k, f_{tail} = tail factor from last observed age to ultimate

Surplus impact of a reserve error via reserve-to-surplus leverage

$E_s = E_r \times \frac{R}{S}$, E_s = surplus error %, E_r = reserve error %, R = carried reserves, S = surplus

Age-to-age link factor on a cumulative triangle

$f_{t \to t+1} = \frac{C_{t+1}}{C_t}$, C_t = cumulative losses at age t, C_{t+1} = cumulative losses at next age; computed only on cumulative data

Average case outstanding diagnostic

$\text{Avg Case O/S} = \frac{\text{Reported}-\text{Paid}}{\text{Open Counts}}$, Reported = cumulative reported losses, Paid = cumulative paid losses, Open Counts = open claim counts

Ceded loss on an excess-of-loss layer

$L_{ceded} = \min\big(\max(L_{gross} - R, 0),\ \ell\big)$, L_{gross} = gross loss, R = retention (attachment), ℓ = layer limit